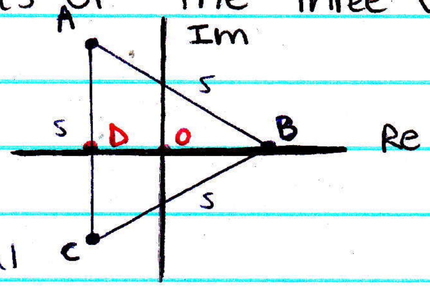
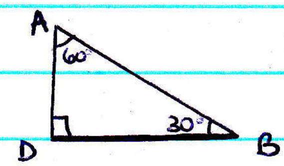
Quiz 3 Solutions

1. In the complex plane, consider the equilateral triangle of side length s, with center at the origin and one point on the positive real axis. Using complex numbers, determine the coordinates of the three vertices

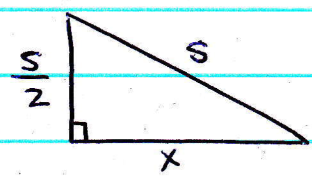
* Draw a picture:



Because we know that this is an equilateral triangle, we also know each angle of the triangle is equal to 60°



If we look at just the top half of the triangle, we have a 30° - 60° - 90° triangle.

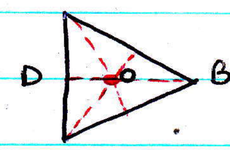


Since $\overline{AC}$ has length s, $\overline{AD}$ is equal to $\frac{s}{2}$. We already know $\overline{AB} = s$, so we only have to find $\overline{DB}$ now.

$$s^2 = (s/2)^2 + x^2$$
$$x^2 = -(s/2)^2 + s^2$$
$$x = \sqrt{s^2 - (s/2)^2}$$
$$x = \sqrt{s^2(1 - 1/4)}$$
$$x = \sqrt{s^2(3/4)} = \boxed{\frac{\sqrt{3}}{2}s}$$

From Assignment #7, we know that the medians of a triangle intersect in a point that is 2/3 the length of each median from each vertex.

Therefore, looking at the first picture, the length of the vertex B to the center (point of intersection) is $\frac{2}{3}$ the length of $\overline{BD}$.



$$\overline{BD} = \frac{\sqrt{3}}{2}s \quad , \text{ so } \quad \overline{BO} = \frac{2}{3}\left(\frac{\sqrt{3}s}{2}\right) = \frac{2\sqrt{3}s}{6} = \frac{\sqrt{3}}{3}s$$

Since point B is on the real axis, its coordinates are $\boxed{\frac{\sqrt{3}}{3}s + 0i}$

$\overline{BO} = \frac{2}{3}BD$. Thus, $\overline{DO} = \frac{1}{3}BD$

$$\overline{BD} = \frac{\sqrt{3}}{2}s \Rightarrow \overline{DO} = \frac{1}{3}\left(\frac{\sqrt{3}}{2}s\right) = \frac{\sqrt{3}}{6}s$$

Point A is on quadrant II. So the x-coordinate must be negative. The length of $\overline{DO} = \frac{\sqrt{3}}{6}s$, so $x = -\frac{\sqrt{3}}{6}s$.

We had already established that $\overline{AD} = \frac{s}{2}$ because it is half the length of $\overline{AC}$. Thus, the y-coordinate, which is on the imaginary axis, is $\frac{s}{2}$.

$$A = \boxed{-\frac{\sqrt{3}}{6}s + \frac{s}{2}i}$$

For Point C, we use the information we already know about the lengths of the different segments:

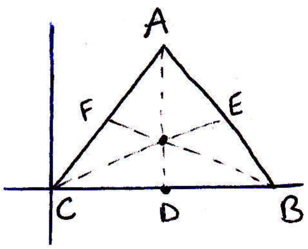
$\overline{AD} = \frac{s}{2}$ so $\overline{CD}$ is also $\frac{s}{2}$

And $\overline{DO} = \frac{\sqrt{3}}{6} s$

Since C is on quadrant III, both x and y coordinates must be negative.

$$C = \boxed{-\frac{\sqrt{3}}{6} s - \frac{s}{2} i}$$

2. Prove that for any equilateral triangle in Euclidean geometry, the line segment from any of the vertices through the center bisects the opposite side.



<u>Given</u>: triangle ABC is equilateral
Let's consider a triangle with one vertex at the origin and one side on the positive real axis.

Since ABC is equilateral, $\overline{AC} = \overline{BC} = \overline{AB} = s$, where s is the side length. The center is equidistant from each of the three vertices. And each angle is equal to $60°$.

If we consider the line segment that goes from point A to the opposide side of the triangle, that line is going to intersect at exactly half the distance from each vertex.

Because ΔACD is a $30°, 60°, 90°$ triangle, $\overline{AD} = \frac{a\sqrt{3}}{2}$ and $\overline{CD} = \frac{a}{2}$. Because $\overline{CD} = \frac{a}{2} = \frac{1}{2}\overline{AC}$, the line from A through the center of the triangle bisects line $\overline{BC}$.

Triangle ACD is similar to triangles ABF and ECD so this is true for all the line segments from the vertices that go through the center of the triangle.